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The discovery 25 years ago of the remarkable objects which came to be known as pulsars, and their identification as neutron stars, fulfilled a prediction made more than 30 years earlier. Over 550 pulsars are now known, almost all detected at radio frequencies. Their pulse periods range from 1.5 ms to several seconds. Most pulsars are single neutron stars but, in an important subset, the pulsar is in a binary orbit with a companion star. Observations have revealed a wealth of detail about the structure and evolution of pulsars and the pulse-emission process, giving new insight into the behaviour of matter in the presence of extreme gravitational and electromagnetic fields. Pulsars have unique properties which make them nearly ideal probes for a wide range of physical studies. Those observational results which are most relevant to these applications are summarized in this paper.

1. Introduction

The discovery of pulsars by Hewish *et al.* (1968) must rank as one of the great events in astronomy. These fascinating objects have opened up new horizons in studies as diverse as quantum-degenerate fluids, relativistic gravity and interstellar magnetic fields. They have provided new avenues for studying binary X-ray sources, the dynamics of globular clusters and the end-points of stellar evolution. The emissions we observe are generated under extraordinary physical conditions, with magnetic and electric fields many orders of magnitude stronger than can be attained in the terrestrial environment.

The outstanding observational characteristic of pulsars is the pulsed emission and its precise periodicity. Figure 1 illustrates the characteristic sharp pulses, of variable amplitude, repeating at regular intervals, in this case, of about 0.714 s. Only an approximate period can be determined from a short train of pulses such as this, but measurements made over longer intervals, many years in some cases, show that the basic pulsational period of pulsars is very stable. This great stability provides the basis for many of the applications of pulsars to studies of physical phenomena.

In contrast, the shape and amplitude of individual pulses are very variable. This shows that the pulses are emitted in a dynamic and rapidly varying environment. However, if one adds a few hundred pulses synchronously with the pulsar period, the resulting mean pulse profile is remarkably stable and has a characteristic shape for each pulsar. Mean pulse profiles for several pulsars are illustrated in figure 2, showing the diversity of shapes observed. Double-peaked profiles are commonly observed and some pulsars have three or more identifiable pulse components.

The pulsar illustrated in figure 1, now known as PSR 0329 + 54, was one of the first pulsars discovered at Cambridge and its period is typical of those early discoveries.

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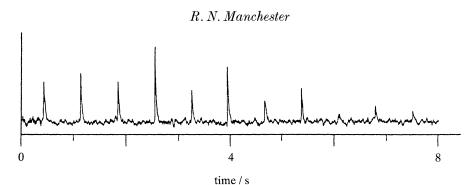


Figure 1. A train of pulses from the strong pulsar, PSR 0329+54, recorded at a radio frequency of 410 MHz (Manchester & Taylor 1977).

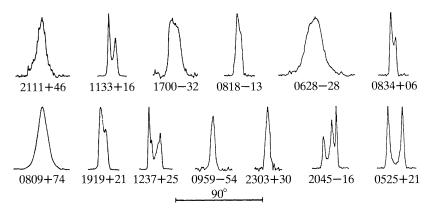


Figure 2. Mean pulse profiles for several pulsars illustrating the diversity of shapes observed. The horizontal bar represents 90° of pulse phase or one quarter of the pulse period (after Manchester & Taylor 1977).

Pulsational periods of the order of seconds allowed several possible models for pulsars including rotation and oscillation of white dwarf stars. However, late in 1968, the situation changed dramatically with the discovery of two short-period pulsars associated with supernova remnants: the Vela pulsar, discovered by Large *et al.* (1968) and the Crab pulsar, discovered by Staelin & Reifenstein (1968). The periods of these pulsars, 89 ms and 33 ms respectively, together with the key discovery by Comella *et al.* (1969) that the period of the Crab pulsar was increasing at a rate of about 36 ns per day, quickly led Gold (1968, 1969) and others to the conclusion that pulsars were rotating neutron stars and that the Crab Nebula was powered by the rotational energy of the pulsar at its centre.

The identification of these pulsars as neutron stars formed in a supernova explosion fulfilled a remarkable prediction by Baade & Zwicky (1934), only two years after the discovery of the neutron, that such stars would exist. It also confirmed the suggestion, made before the discovery of pulsars by Pacini (1967), that the Crab Nebula was powered by a spinning and highly magnetized neutron star.

The rotating neutron star model has stood the test of time and is now firmly established as the basis of the pulsar phenomenon. Discoveries such as the first binary pulsar (Hulse & Taylor 1975) and the first millisecond pulsar (Backer *et al.* 1982) have only served to reinforce its validity. Beams fixed to the rotating star sweep across the sky and, if they pass across the Earth, a pulse may be observed.

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Pulse shape and polarization measurements strongly suggest that beams are emitted more or less radially from the vicinity of a magnetic pole on the star (Radhakrishnan et al. 1969). In some pulsars, with the Crab pulsar being the most notable example, two pulses per period are emitted. The second pulse, or interpulse, is believed to be emitted from the opposite magnetic pole to the first or main pulse.

Neutron stars are incredibly dense objects, with mass comparable with that of the Sun, but radius of only 16 km or so. Their high density and consequent strong selfgravitation enables them to withstand rotation rates as high as several hundred times a second as is required by the observation of millisecond pulsars. They are believed to have a solid outer crust and an interior which largely consists of degenerate neutron superfluid. Either by flux conservation during their formation, or by subsequent generation, neutron stars possess extremely strong magnetic fields, of the order of 10^8 T in typical pulsars. These fields are frozen into the neutron star crust and hence rotate with the star, generating intense electric fields and accelerating charged particles to ultrarelativistic speeds. These charged particles, in turn, generate the emission beams that we observe as pulses. Despite their intensity, these emission beams represent only a small fraction of the total energy-loss budget. Most of the energy lost to the system is carried away by charged particles or low-frequency electromagnetic waves.

In the remainder of this paper, I shall outline the principal observed properties of pulsars, emphasizing those characteristics which are most relevant to the application of pulsars to studies in physics. More extensive descriptions can be found in the monographs by Manchester & Taylor (1977) and Lyne & Graham-Smith (1990).

2. The present sample

There are currently about 550 pulsars known. All but five of these lie within our Galaxy; the five lie in our nearest-neighbour galaxies, the Magellanic Clouds. As shown in figure 3, pulsars are concentrated along the Galactic equator, the Milky Way, but with a considerable spread in latitude. Much of this spread can be attributed to the fact that pulsars are high-velocity objects, with typical velocities of the order of 200 km s⁻¹; because of this they move far from their birthplace during their active lifetime. All surveys for pulsars are sensitivity limited, and lowluminosity pulsars can only be detected if they lie relatively close to the Sun. Consequently, we detect only a small fraction of all active pulsars in the Galaxy; the total number is estimated to be more than 10^5 (Lyne *et al.* 1985; Narayan & Ostriker 1990).

Pulsars fall into two rather distinct groups: relatively young isolated pulsars, typically with periods between 0.1 and 1 s, and the short-period, but very old, 'millisecond' pulsars. Most of the known millisecond pulsars are found in globular clusters, dense groups of stars lying in the halo of our Galaxy, and more than half of them are in binary orbits with other stars. In contrast, only about 1% of the 'normal' pulsars are members of binary systems. These relationships are illustrated in figure 4.

Although pulsar periods are very stable, they are not constant. All pulsars lose rotational energy to some combination of accelerated particles and low-frequency magnetic-dipole radiation and, hence, slow down. For a dipole magnetic field, the braking torque is given by

$$N = I\Omega' = -\left(2/3c^3\right) \left(B_0^2/R_0^6\right) \Omega^3,$$

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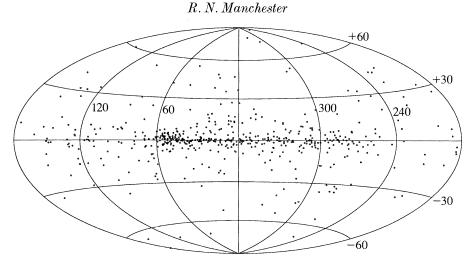


Figure 3. Distribution of known pulsars in galactic coordinates. The central horizontal line is the galactic equator with the galactic centre at the centre of the figure. Degrees of latitude and longitude are indicated. The concentration of pulsars centred around longitude 50° reflects the higher sensitivity of pulsar searches conducted at Arecibo compared with those at other observatories.

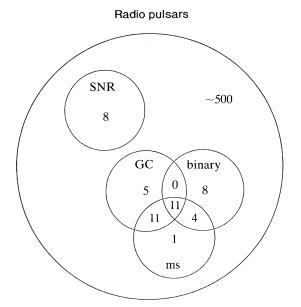
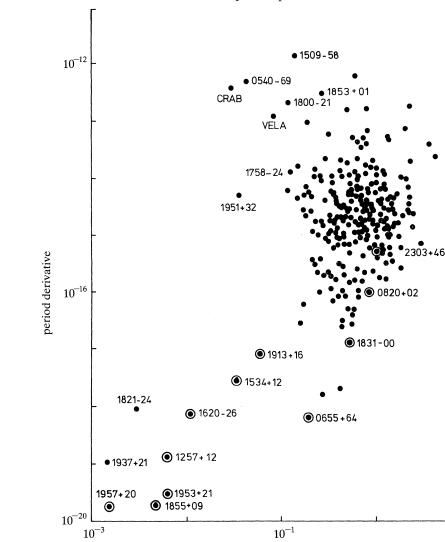


Figure 4. Venn diagram illustrating the different classes of pulsars. Most pulsars are isolated and relatively young; eight of the youngest are associated with supernova remnants. The millisecond pulsars are often in binary systems with other stars and most of those known are in globular clusters.

where I is the neutron-star moment of inertia, $\Omega = 2\pi/P$ is its angular rotation frequency and $\Omega' = 2\pi P'/P^2$ is its first time derivative, B_0 is the magnetic flux density at the neutron-star surface and R_0 is the neutron-star radius. Hence, for a constant magnetic field, and assuming that the spin rate at birth was much greater than its present value, the age of a pulsar is given by $\tau = P/(2P')$, and its surface magnetic field B_0 is proportional to $(PP')^{\frac{1}{2}}$.

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period / s Figure 5. Plot of period derivative (in dimensionless units) against period for most of the known pulsars. Young pulsars associated with supernova remnants are clustered in the top left of the figure, whereas millisecond pulsars are located in the bottom left. Pulsars which are members of a binary system are indicated by a circle around the point.

Most pulsars now have measured period derivatives. Figure 5 shows that, for 'normal' pulsars, the derivative is typically about 10^{-15} . For a period of one second, the corresponding surface magnetic field strength is about 10^8 T and the age is about 10^6 years. As young pulsars age, they move diagonally down into the pool of normal pulsars. It is clear that millisecond pulsars, which have magnetic field strengths three to four orders of magnitude smaller than normal pulsars, cannot be formed by simple ageing of young pulsars. A clue to their probable formation mechanism is given by the large proportion which are members of binary systems.

Millisecond pulsars are believed to be old neutron stars which have been 'recycled' by being spun up as a result of accretion from an evolving binary companion (Smarr

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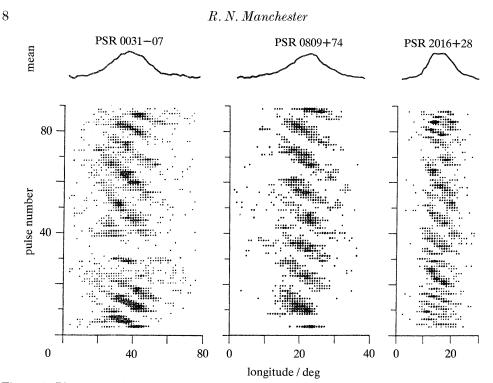


Figure 6. Phase-time diagrams for three pulsars exhibiting the drifting subpulse phenomenon. Each line represents one pulse, and successive lines are aligned in pulse phase according to the pulsar period. Time increases to the right and upward and the mean pulse profile for each pulsar is shown at the top of the figure (Manchester & Taylor 1977).

& Blandford 1976). Accretion of matter onto the neutron star also results in the emission of X-rays and, in cases where there is a modulation related to the neutronstar rotation, this spin-up can be directly observed. The high space density of stars in the cores of globular clusters makes capture of a neutron star by an evolving ordinary star quite probable, and so such X-ray binary systems and their products – millisecond pulsars – are relatively common in these clusters (Manchester *et al.* 1991). The formation and evolution of binary and millisecond pulsars is extensively discussed in the recent review by Bhattacharaya & van den Heuvel (1991).

3. Properties of the pulsed emission

As mentioned in §1, the shape and intensity of individual pulses varies greatly from pulse to pulse. Most pulses consist of one or two 'subpulses' which are narrow and have a relatively simple profile, in contrast to the mean pulse profiles which often have several distinct components (figure 2). In general, these components occur at pulse phases (or longitudes) where there is a more frequent occurrence of subpulses; the amplitude of subpulses tends to be less variable than their frequency at a given longitude. In most pulsars the central longitude of subpulses varies from pulse to pulse in a random fashion, but there exists a class of pulsars in which there is a systematic 'drift' of subpulse longitude from one pulse to the next. Three examples of such 'drifting-subpulse' pulsars are shown in figure 6. In most but not all cases, the subpulses drift from the rear of the integrated profile to the front.

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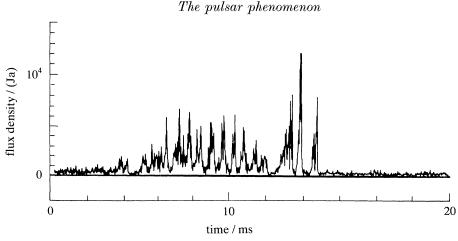


Figure 7. A single pulse from PSR 0950+08 (111.5 MHz) observed with time resolution of 28 μ s showing the micropulse structure. This example is somewhat atypical in that the micropulse amplitudes have a quasi-periodic modulation (after Hankins 1971).

Over most of the radio band, the emission from pulsars has a steep non-thermal spectrum, that is, it is weaker at higher frequencies. Typical spectral indices α , where the observed flux density $S \approx \nu^{\alpha}$, are -1.5 for normal pulsars and -2.5 for millisecond pulsars. A few young pulsars, most notably the Crab and Vela pulsars, are detectable at optical and higher energies. The spectrum of this high energy emission is not continuous with the radio spectrum, indicating that different emission mechanisms operate in the two régimes.

With observations of higher time resolution, the emission from most, but perhaps not all, pulsars is seen to be modulated on timescales of 10–100 µs, one or two orders of magnitude shorter than the typical subpulse timescale. An example is illustrated in figure 7. These 'micropulses' are probably the elemental units of pulsar emission, and their short timescale puts strong limits on the size of the emission region. The observed radio flux density from pulsars is typically about 10^{-26} W m⁻² Hz⁻¹ and most pulsars are at distances of about 10^{20} m from the Earth, so the spectral luminosity is about 10^{14} W Hz⁻¹. Pulse duty cycles are typically about 0.1, suggesting that the radiation is emitted into a solid angle of about a steradian. Both the observed micropulse timescale and the magnetic-pole model for pulsar emission imply that the pulse emission region is not more than about 10^4 m across, so the specific intensity I_{ν} of the radio emission is about 10^6 W m⁻² Hz⁻¹ ster⁻¹. In the Rayleigh–Jeans limit, this corresponds to a brightness temperature

$$T_{\rm b} = I_{\nu} \lambda^2 / (2k)$$

of more than 10^{28} K. Clearly, the radio emission cannot be thermal; coherent processes such as maser emission are required. In the optical and higher frequency régimes, brightness temperatures are not so high and incoherent emission is possible.

Besides its high brightness temperature, another important feature of the radio emission from pulsars is its high polarization. As figure 8 illustrates, in some pulsars, mean pulse profiles are essentially 100% linearly polarized. This implies that every individual pulse is likewise 100% polarized. Circular polarization is also observed, but usually with a lower fractional polarization. The position angle of the linearly polarized component generally varies smoothly through the pulse, sometimes by as much as 180°, with the form of the variation usually consistent with emission from

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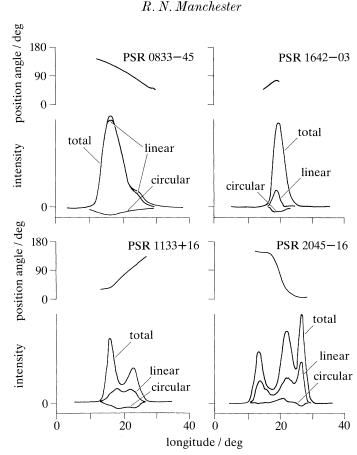


Figure 8. Mean pulse profiles for four pulsars showing the variations of linearly and circularly polarized emission through the pulse. The upper curve for each pulsar is the position angle of the linearly polarized component (Manchester & Taylor 1977).

the vicinity of a magnetic pole (Radhakrishnan *et al.* 1969; Lyne & Manchester 1988). Exceptions to this smooth position-angle variation do occur, but observations of the polarization of individual pulses (Stinebring *et al.* 1984) show that these generally can be attributed to the occurrence of two orthogonally polarized emission modes at a given pulse longitude.

4. Pulsars and the interstellar medium

The pulsed nature of the emission from pulsars, its often high linear polarization and the very small size of the emission region, make pulsars unique probes of the interstellar medium. Dispersion is the most obvious effect of the interstellar medium on the observed pulses. Much of the gas in the interstellar medium is ionized and the group velocity for propagation of a radio wave through this plasma is a function of its frequency. With respect to propagation at infinite frequency, the delay in reception of a pulse at frequency ν is proportional to $M_{\rm D} \nu^{-2}$, where the dispersion measure $M_{\rm D}$ is defined by

$$M_{\rm D} = \int_{\rm path} n_{\rm e} \,\mathrm{d}l.$$

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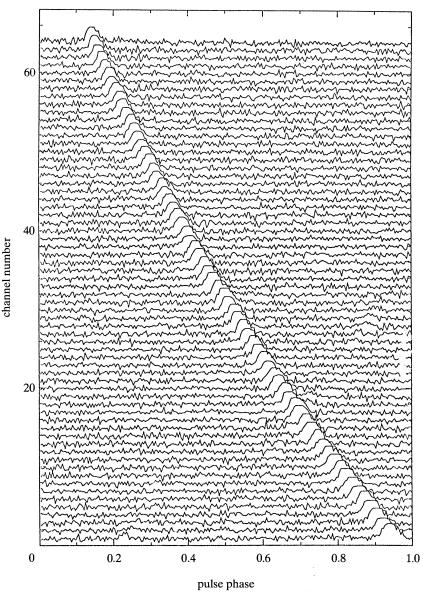


Figure 9. Dispersive delay of the pulsed signal from PSR 1641 - 45, which has a dispersion measure of 475 cm^{-3} pc, from data recorded at the Parkes Observatory of the ATNF. Each line represents the integrated profile from one channel of a 64-channel filterbank; channel 1 corresponds to a radio frequency of 1400 MHz and channel 64 to 1720 MHz. Since the pulsar period is about 455 ms, the differential delay across this band is approximately 365 ms.

The second-order frequency dependence of the dispersion delay is clearly visible in figure 9. This delay is easily measurable. Hence, if the distance to the pulsar is known, a value for the mean free electron density in the path to the pulsar is obtained. Alternatively, if the distribution of free electrons in the galactic disc is known, the dispersion measure gives a value for the distance to the pulsar.

Interstellar magnetic fields result in Faraday rotation of the plane of polarization of a radio signal. (The strong magnetic fields associated with the pulsar itself produce

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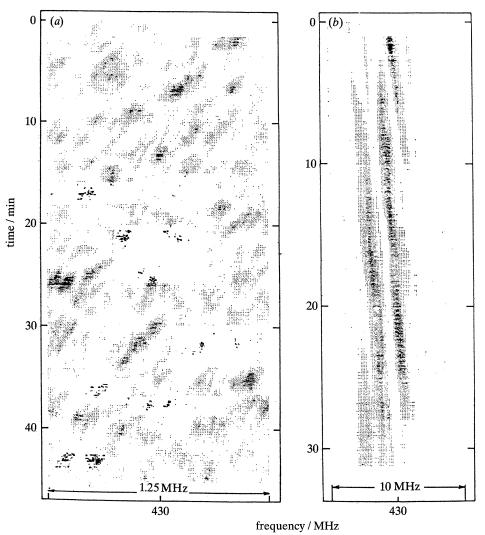


Figure 10. Signal intensity versus radio frequency and time for two pulsars showing variations due to interstellar scintillation: (a) PSR 1737+13, (b) PSR 0823+26 (Cordes *et al.* 1985).

no detectable rotation.) The high linear polarization of the pulses from many pulsars makes this effect relatively easy to measure. Again, with respect to propagation at infinite frequency, the rotation in the plane of polarization is proportional to $M_{\rm R} \nu^{-2}$. The rotation measure is related to the component of the magnetic field parallel to the line-of-sight B_{\parallel} by

$$M_{\rm R} = \int_{\rm path} n_{\rm e} B_{\parallel} \, {\rm d}l. \label{eq:math}$$

Hence the mean parallel component of the field $\langle B_{\parallel} \rangle$, weighted by the local electron density, is simply proportional to $M_{\rm R}/M_{\rm D}$. This technique has been used to investigate the structure of the galactic magnetic field (Lyne & Smith 1989) and is valuable, since direct estimates of interstellar magnetic flux densities are difficult to obtain by other means.

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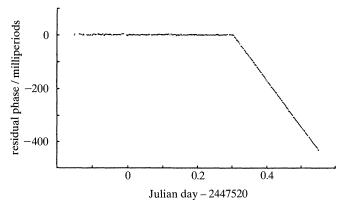


Figure 11. Phase residual, that is, the difference between the observed and predicted arrival time of a pulse, for the Vela pulsar (PSR 0833-45) over a 12 h period on 24 December 1988 (McCulloch *et al.* 1990). The sudden change of slope corresponds to a decrease in the pulsar period of about 160 ns or 1.8 parts in 10^6 .

Small-scale fluctuations in the interstellar electron density distribution result in multipath propagation and hence scintillation of the received radio signal. This is most clearly seen in plots of signal intensity against both frequency and time, such as those shown in figure 10. In some pulsars at some times, the variations have systematic structure such as that shown for PSR 0823 + 26, whereas other pulsars show more random scintillations. The small size of the emission region makes pulsars effective probes of these phenomena.

5. Pulsar timing

Careful measurements of pulse arrival times over long intervals, decades in some cases, show that pulsar periods are incredibly stable. As an illustration of this we quote the period of the first known millisecond pulsar, PSR 1937 + 21, which was

$1.557\,806\,448\,872\,75\,\,\mathrm{ms}\pm0.000\,000\,000\,000\,03\,\,\mathrm{ms}$

on 29 November 1982, at 1903 UT (Rawley *et al.* 1988). The precision of this measurement is limited by the precision of terrestrial time standards, opening up the possibility that pulsars may be used to define terrestrial time, at least over long time intervals.

Although the period stability of pulsars is very high, it is not perfect, especially for young pulsars. Such pulsars are subject to 'glitches', sudden and unpredictable changes in period. An example of a glitch in the period of the Vela pulsar is illustrated in figure 11. The Vela pulsar has suffered nine glitches over 23 years of observation. Other young pulsars, for example, PSR 1737-30 (McKenna & Lyne 1990), have more frequent glitches, but none has been detected in the older millisecond pulsars. Glitches are believed to result from sudden changes in the interior superfluid of the neutron star and hence makes studies of this exotic state of matter possible.

The remarkable stability of pulsar periods, particularly for the millisecond pulsars, allows many interesting effects to be measured. For example, observations over several years give the change in position, or proper motion, of a pulsar due to its space velocity. For millisecond pulsars, such measurements can have a precision of better than one milliarcsecond per year (Ryba & Taylor 1991). These and other

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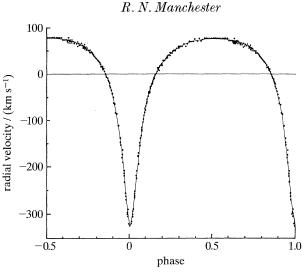


Figure 12. Variation of the radial velocity of PSR 1913 + 16 during its orbit (Hulse & Taylor 1975). The pulsar period is 59 ms, the orbital period is $7^{h} 45^{m}$, and the orbital eccentricity is 0.617.

measurements of pulsar proper motion show that pulsars are high-velocity objects, typically with velocities of the order of 200 km s⁻¹. These velocities are believed to originate at the birth of the neutron star, probably as a result of an asymmetry in the supernova explosion.

But the most remarkable results have come from the study of binary pulsars. Figure 12 shows the orbital velocity variations for the first binary pulsar discovered, PSR 1913+16. The orbit is highly eccentric. At periastron, the pulsar velocity exceeds 0.1% of the velocity of light and it approaches to within one solar radius of its companion. These parameters imply that relativistic perturbations to the orbital motion are likely to be detectable, and indeed five different 'post-keplerian' effects have now been observed in this pulsar (Taylor *et al.* 1992) allowing unprecedented tests of the validity of theories of relativistic gravity. Searches for millisecond pulsars at several observatories around the world have now detected several other binary millisecond pulsars which show relativistic perturbations to their orbital motion.

Another remarkable result to come from pulsar timing is the first detection of extra-Solar-System planetary companions to a star. The millisecond pulsar PSR1257 + 12 has two companions of 3.4 and 2.8 Earth masses with orbital periods of about two and three months, respectively (Wolszczan & Frail 1992). This discovery opens up the possibility of using pulsars as tools for studying the formation of planetary systems.

6. Conclusions

It is clear, even from this brief summary, that pulsars have unique characteristics which make possible the study of a wide range of physical phenomena. They really are remarkable laboratories for the study of physics.

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